1.1 Separation of scales: Why complex systems need a new mathematics

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One of the central insights about complex systems is that the effect of dependencies among components cannot be fully represented by traditional mathematical and conceptual approaches. A key to their limitation is that they are applicable only to systems in which there is a separation of behavior between the micro and macro scales. Interactions among the parts that cause behaviors across scales violate this separation.

Consider a block sliding down an inclined plane. In a traditional approach, micro and macro scales are treated separately. To address dynamics at the micro scale—the molecules—we average over them and, using thermodynamics, describe their temperature and pressure. To address dynamics at the macro scale—the motion of the block on the inclined plane—we use Newtonian physics to talk about their large scale motion (see Fig. 1). In this case, the pieces can be considered to be acting either independently, like the random relative motion on the micro scale, or coherently, like the average motion on the macro scale. Since the scales are sufficiently distinct, separated by orders of magnitude, we do not encounter a problem in describing them separately. Finally, often unstated, the structures of the block and the plane are considered fixed.

Thus, traditionally, there were three aspects of a system: fine scale, dynamic, and fixed. A glass of water on a table with an ice cube in it might be treated by considering the movement and melting of the ice cube, the average over molecular vibrations, and the fixed structure of the glass. At longer time scales, the water will evaporate, the glass will flow, the table may rot, but this is not important at a particular scale (or a range of scales) of observation.

Consider the earth viewed from space. The earth is highly complex. Still, we can describe it as a planet orbiting the sun in a predictable fashion. Most of the details of what happens on Earth play no role at the scale of its orbit. For the earth, at the orbital scale, all the internal structure can be averaged to a point. The bodies of the solar system are assumed unchanging and the material of each of them is separated from other solar objects. The dynamic behavior can then be modeled and predicted.

When separation of scales works, we can describe not only the system as it exists in isolation, but also how it responds to external forces. Forces that act on the earth at the scale of orbital motion couple to the dynamic behavior that occurs at that scale. Thus if we were to consider a new celestial body entering the solar system, unless it disrupted the structure of the system (i.e. by shattering a planet) and as long as we continue to be interested in the scale of orbital motion, we can describe the behavior of the system using these same degrees of freedom.

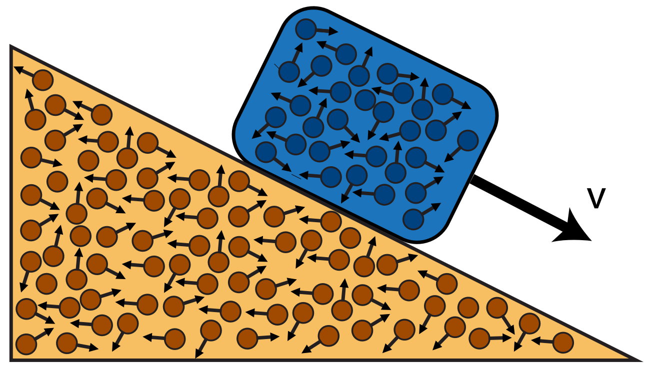

FIG. 1: Schematic diagram of a block (with a velocity at a particular moment, v) sliding down an inclined plane. The macroscopic motion subject to gravity and friction may be treated using Newton's laws of motion, while the microscopic behavior of the atoms may be treated using thermodynamics by considering the local oscillations of groups of atoms as random and independent (the probability that one group is in a particular state is independent of the state of another group); the statistical treatment of that movement leads to the determination of pressure and temperature of the block and the inclined plane.

For complex systems, it is still true that the questions we most want to answer have to do with the larger scale information. Significantly, the scale of description and scale of interactions are similar. When we have a description of the larger scale behavior we are also considering the larger scale impacts of the environment on the system and reciprocally.

But many systems, especially those we are interested in understanding and influencing, are not well described by separate micro and macro scales. Consider a flock of birds. If all of the birds flew independently in different directions, we would need to describe each one separately. If they instead all went in the same direction, we could simply describe their average motion. However, if we are interested in their movement as a flock, describing each bird's motion would be too much information and describing the average would be too little information. Similarly, for traffic jams, market behavior and weather, the average behavior is not enough and all the details are too much to be useful. Understanding complex behavior that is neither independent nor coherent behavior is best described across scales. This requires knowing which information can be observed at a scale of interest.